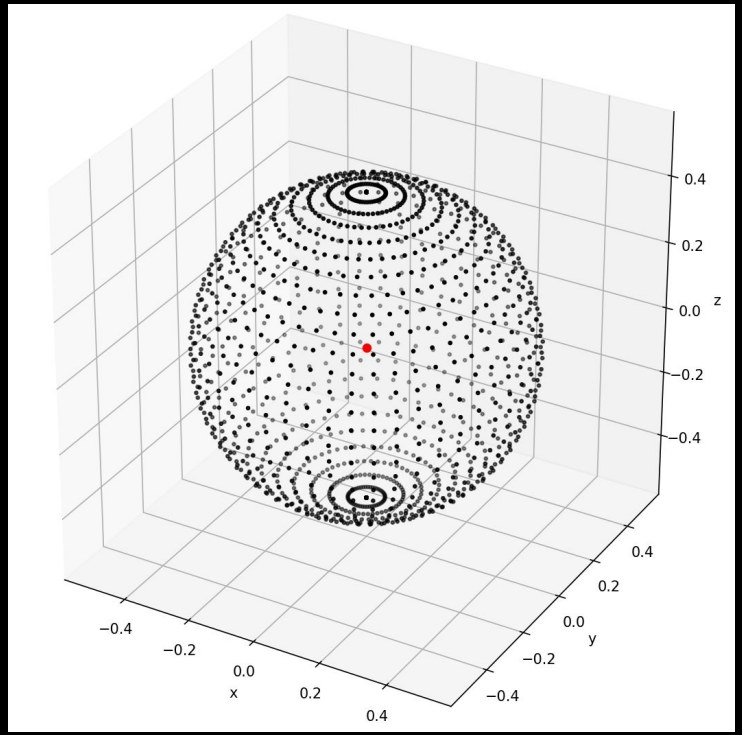
Apparent Horizon Finder



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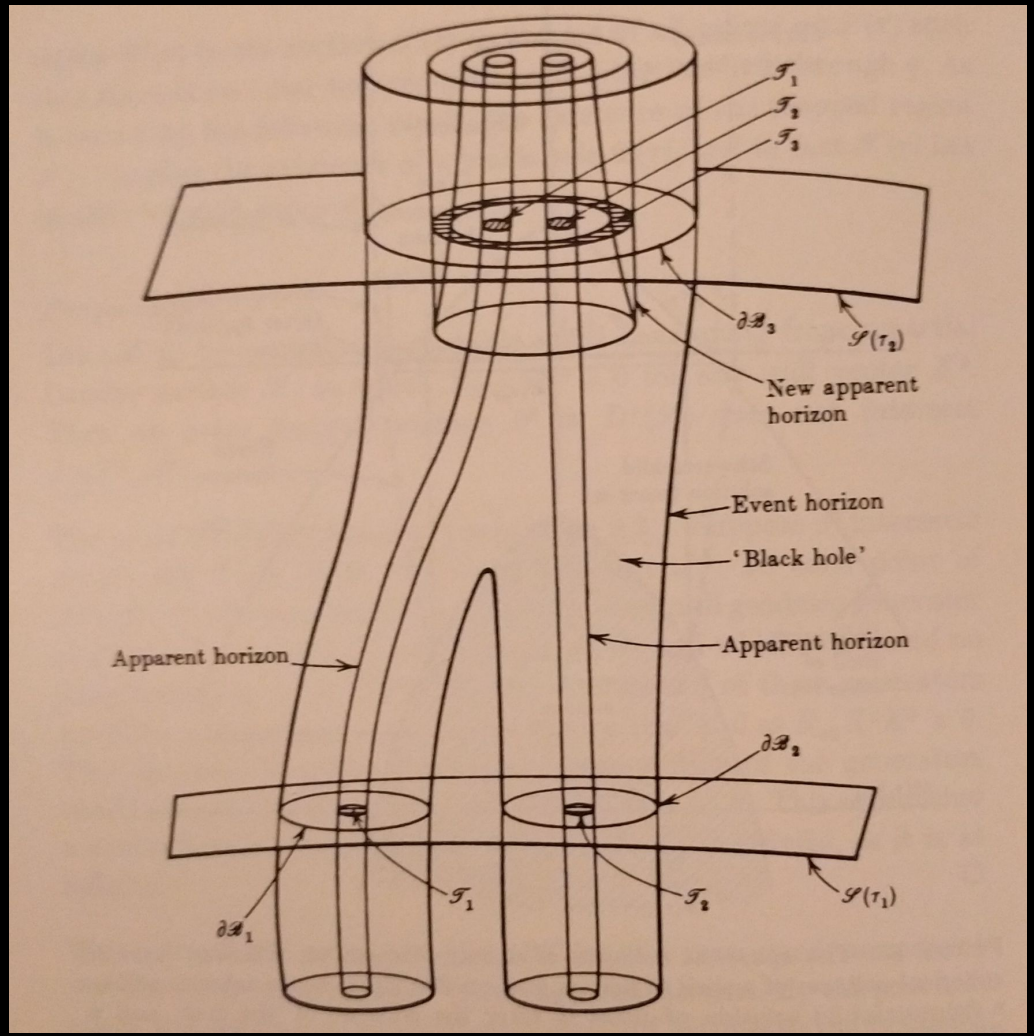


- Short version: an apparent horizon (AH) is the outermost 2D trapped surface, where the "expansion", Θ, is zero
- This is the surface where the area of a spherical flash of light rays emitted radially outwards will remain constant.
- Apparent horizon is inside the event horizon. Coincides in static case
- In binaries, a 3rd and 4th AH appear when merging

Numerical Methods for Star-shaped AHs

GRchombo's AHFinder discretizes the 2D AH surface and uses a quasi-Newton method from the PETSc library to find the zero of the expansion for star-shaped horizon, given some initial guess.

Newton's Method (1D):

$$f_{n+1} = f_n - \gamma \left(\Theta'\left(f_n\right)\right)^{-1} \Theta\left(f_n\right) , \quad 0 < \gamma \leq 1$$

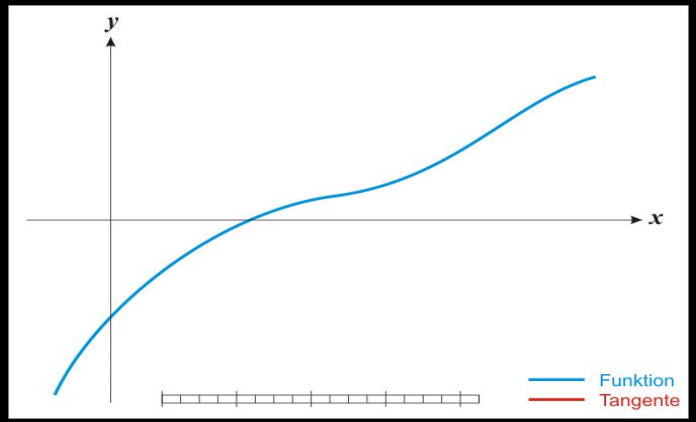
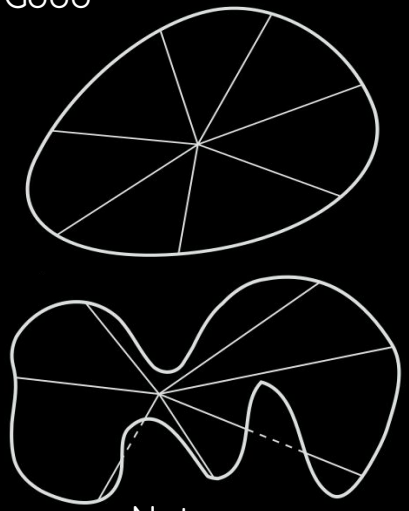


Image from Wikipedia

Good



Not so easy

Images from M. Alcubierre, *Introduction to 3+1 Numerical Relativity*

AHFinder Class

How to use?

1) Install PETSc and change Chombo's Make.defs.local -> see full slides

2) Change your Example -> see full slides

3) Add parameters to 'params.txt' file

4) What is the output?

Quick version: consult BinaryBH / KerrBH Examples as a reference

AHFinder – How to use

3) Add parameters to ‘params.txt’ file

params.txt

```
#Apparent Horizon finder
AH_activate = 1
AH_num_ranks = 65
AH_num_points_u = 65
AH_num_points_v = 48
#AH_solve_interval = 1
#AH_print_interval = 1
#AH_track_center = true
#AH_predict_origin = true
#AH_level_to_run = 0
#AH_start_time = 0.
#AH_give_up_time = -1.
```

...

```
#AH_merger_search_factor = 1.
#AH_merger_pre_factor = 1.
#AH_allow_re_attempt = 0
#AH_max_fails_after_lost = -1
#AH_verbose = 1
#AH_print_geometry_data = 0
#AH_re_solve_at_restart = 0
#AH_stop_if_max_fails = 0

#AH_1_initial_guess = 0.3
#AH_2_initial_guess = 0.3

#AH_num_extra_vars = 2
#AH_extra_vars = chi d1_Ham d2_A11

AH_set_origins_to_punctures = 1
```

- params.txt: there are many AH parameters. The commented values are the default values. Consult AHParams.hpp for more information (meaning and default values).

AHFinder – How to use

4) What is the output? Part I - Command Line


```
BinaryBH_000172.3d.hdf5  coords_AH2_0022.out  coords_AH3_0071.out  coords_AH3_0131.out
BinaryBH_000176.3d.hdf5  coords_AH2_0023.out  coords_AH3_0072.out  coords_AH3_0132.out
BinaryBH_000180.3d.hdf5  coords_AH2_0024.out  coords_AH3_0073.out  coords_AH3_0133.out
BinaryBH_000184.3d.hdf5  coords_AH2_0025.out  coords_AH3_0074.out  coords_AH3_0134.out
BinaryBH_000188.3d.hdf5  coords_AH2_0026.out  coords_AH3_0075.out  coords_AH3_0135.out
BinaryBH_000192.3d.hdf5  coords_AH2_0027.out  coords_AH3_0076.out  coords_AH3_0136.out
BinaryBH_000196.3d.hdf5  coords_AH2_0028.out  coords_AH3_0077.out  coords_AH3_0137.out
BinaryBH_000200.3d.hdf5  coords_AH2_0029.out  coords_AH3_0078.out  coords_AH3_0138.out
coords_AH1_0000.out      coords_AH3_0019.out  coords_AH3_0079.out  coords_AH3_0139.out
coords_AH1_0001.out      coords_AH3_0020.out  coords_AH3_0080.out  coords_AH3_0140.out
coords_AH1_0002.out      coords_AH3_0021.out  coords_AH3_0081.out  coords_AH3_0141.out
coords_AH1_0003.out      coords_AH3_0022.out  coords_AH3_0082.out  coords_AH3_0142.out
coords_AH1_0004.out      coords_AH3_0023.out  coords_AH3_0083.out  coords_AH3_0143.out
coords_AH1_0005.out      coords_AH3_0024.out  coords_AH3_0084.out  coords_AH3_0144.out
coords_AH1_0006.out      coords_AH3_0025.out  coords_AH3_0085.out  coords_AH3_0145.out
coords_AH1_0007.out      coords_AH3_0026.out  coords_AH3_0086.out  coords_AH3_0146.out
coords_AH1_0008.out      coords_AH3_0027.out  coords_AH3_0087.out  coords_AH3_0147.out
```

```
pout.88
pout.89
pout.9
pout.90
pout.91
pout.92
pout.93
pout.94
pout.95
pout.96
pout.97
pout.98
pout.99
slurm-17662242.out
stats_AH1.out
stats_AH2.out
stats_AH3.out
```

- Output will be a ‘coords’ file for each AH and for each step, containing to coordinates of the AH surface, and a ‘stats’ file for each AH, containing convergence information (e.g. area and spin) for all timesteps.

AHFinder – How to use

4) What is the output? Part II - ‘coords’ files

coords_AH1_000000.dat

# theta	phi	r	chi	dx_chi	dy_chi	dz_chi
0.0000000000	0.0000000000	0.4839762884	5.5056177192e-02	7.1150767569e-18	7.1015242298e-18	2.2786411876e-01
0.3141592654	0.0000000000	0.4839941742	5.5085085743e-02	7.0672731933e-02	-8.8091426514e-20	2.1664488191e-01
0.6283185307	0.0000000000	0.4840507182	5.5149123468e-02	1.3438728357e-01	6.6136332522e-18	1.8441576914e-01
0.9424777961	0.0000000000	0.4841351993	5.5221840971e-02	1.8513333988e-01	2.6969529041e-18	1.3407117896e-01
1.2566370614	0.0000000000	0.4842168287	5.5275258124e-02	2.1776052981e-01	4.6349642874e-18	7.0594833460e-02
1.5707963268	0.0000000000	0.4842598993	5.5293492230e-02	2.2914994366e-01	3.8556939759e-18	-2.4070510568e-18
1.8849555922	0.0000000000	0.4842168289	5.5275258178e-02	2.1776052986e-01	2.7240579584e-18	-7.0594833478e-02
2.1991148575	0.0000000000	0.4841351998	5.5221841092e-02	1.8513333999e-01	5.1228552650e-18	-1.3407117903e-01
2.5132741229	0.0000000000	0.4840507191	5.5149123687e-02	1.3438728371e-01	6.3696877634e-18	-1.8441576933e-01
2.8274333882	0.0000000000	0.4839941758	5.5085086123e-02	7.0672732060e-02	5.6785088784e-18	-2.1664488230e-01
3.1415926536	0.0000000000	0.4839762899	5.5056177544e-02	5.2177229551e-18	5.2041704279e-18	-2.2786411914e-01
0.0000000000	0.3141592654	0.4859051601	5.5495880920e-02	5.3803532810e-18	5.3668007538e-18	2.2833166757e-01
0.3141592654	0.3141592654	0.4870958848	5.5783990070e-02	6.7444169396e-02	2.1909965817e-02	2.1735972065e-01

- ‘coords’ files: contain the coordinate system information about the surface of the AH (spherical coordinates - theta, phi, r - above).
  With the parameters ‘AH_num_write_vars = 2’ and ‘AH_write_vars = chi d1_chi’ the example above also outputs the value of ‘chi’ and its derivatives at each point of the horizon. These can be diagnostic variables and include 1st or 2nd derivatives.

AHFinder – How to use

4) What is the output? Part III - stats_AH1.dat


```
head stats_AH1.dat
#          time                file                area                mass          irreducible mass                spin          dimless spin-x          dimless spin-y
   2.0000000000    4.0000000000e+00    1.6418300976e+01    5.7151703384e-01    5.7151703359e-01    3.3947220719e-05    1.0990679865e-08   -5.9239748341e-05
   2.5000000000    5.0000000000e+00    1.7487064429e+01    5.8982547674e-01    5.8982547673e-01    6.7097940928e-06   -5.1749065646e-07   -1.0211252952e-05
   3.0000000000    6.0000000000e+00    1.8423481166e+01    6.0541185337e-01    6.0541185326e-01    2.2251144079e-05    4.3507303577e-07   -3.5360014956e-05
   3.5000000000    7.0000000000e+00    1.9195748854e+01    6.1797029939e-01    6.1797029828e-01    7.4045531728e-05    1.3623533919e-07   -1.1964564076e-04
```

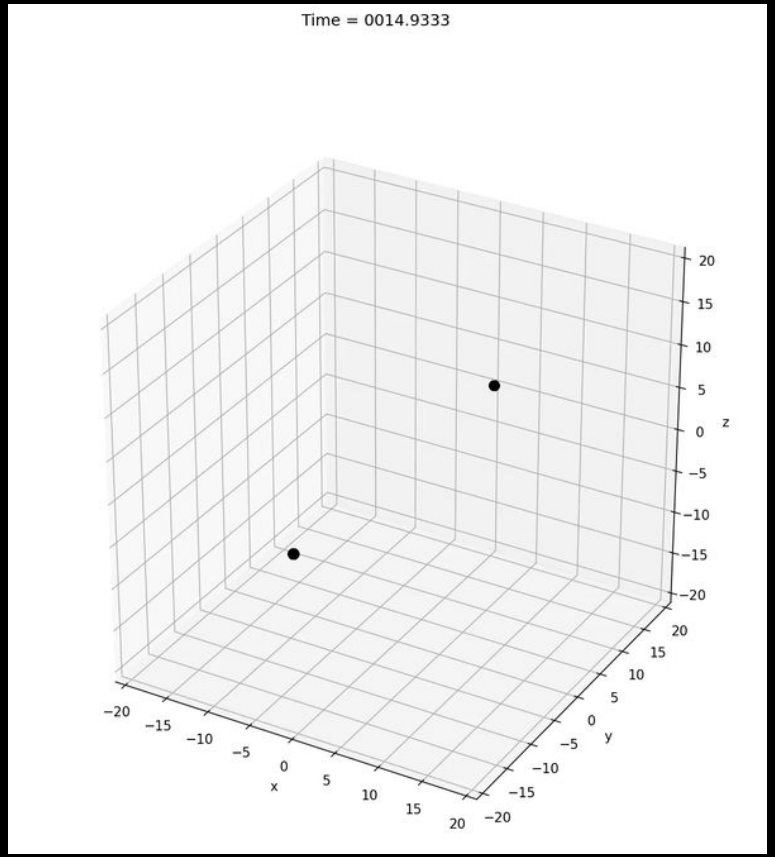
...

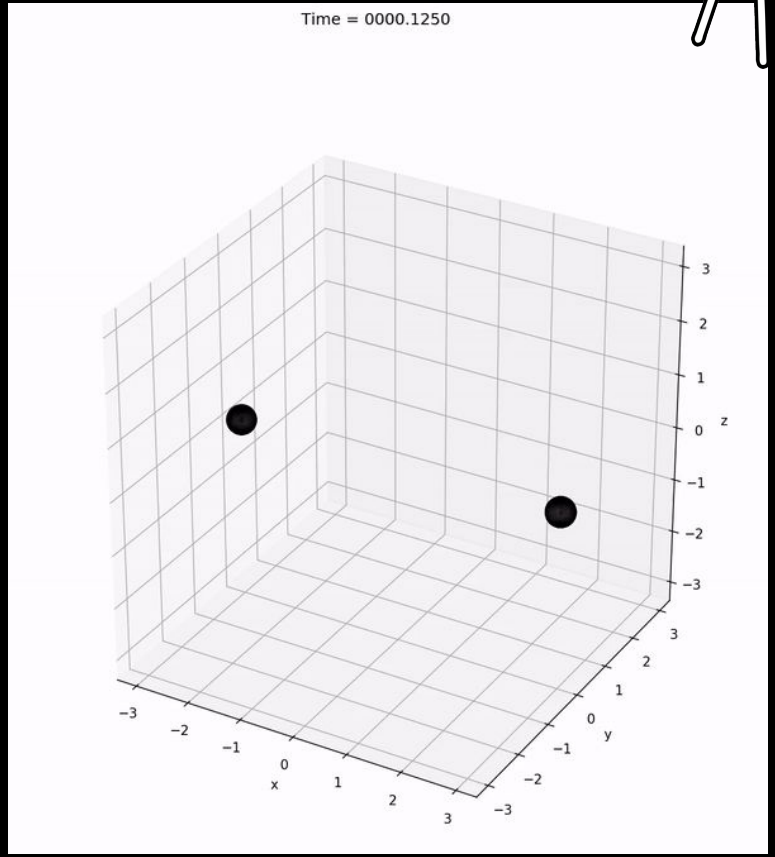
```
...  dimless spin-z  dimless spin-z-alt          origin_x          origin_y          origin_z          center_x          center_y          center_z
     4.3389940102e-06    0.0000000000e+00    6.1561971694e+00    7.9999998311e+00    6.2348738671e-23    6.1468078594e+00    8.0000049371e+00    0.0000000000e+00
    -4.9873397381e-06    0.0000000000e+00    6.2283372624e+00    7.9999997328e+00    1.0867085297e-22    6.2343423711e+00    7.9999910394e+00    0.0000000000e+00
    -1.0015825320e-05    0.0000000000e+00    6.3160466820e+00    7.9999994787e+00    7.1447725409e-23    6.3162350431e+00    7.9999853858e+00    0.0000000000e+00
     6.4701339056e-06    0.0000000000e+00    6.4135201723e+00    7.9999991857e+00    9.7917505011e-23    6.3855359687e+00    8.0000071376e+00    0.0000000000e+00
```

- 'stats' files: print the area, spin and center of each found AH at each timestep
- 'origin' is the origin of the coordinate system used in the 'coords' file.
- 'center' is an approximate geometric center of the surface found (an approximate location of the puncture).

AHFinder – How to use

4) What is the output? When it goes right...





Memories

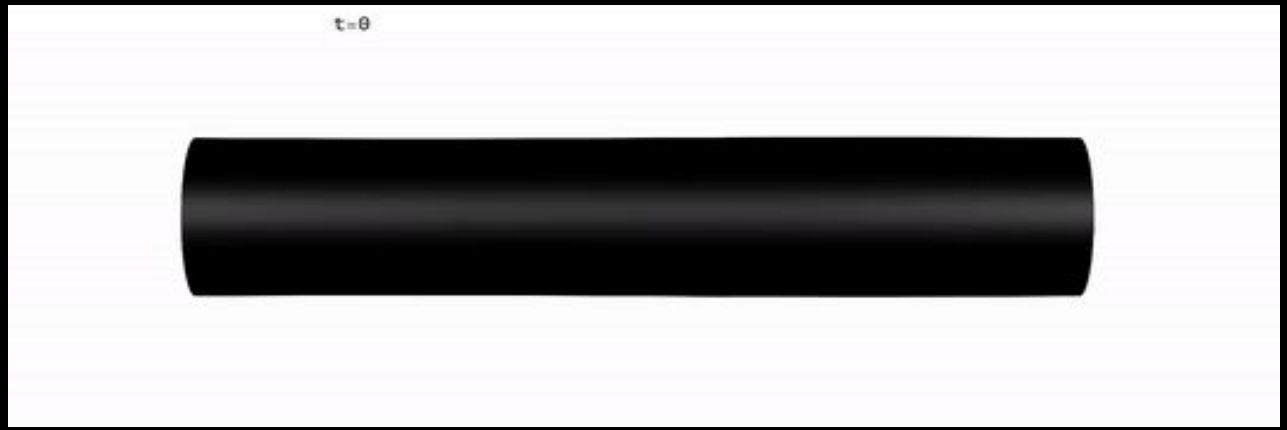
```
145         for (int u = m_umin; u < m_umax; ++u)
146         {
147             #if CH_SPACEDIM == 3
148             double& ff = f[v][u];
149             #else
150             double &ff = f[u];
151             #endif
152
153                     ff = sqrt(m_inital_guess);
154         }
```

Setting initial radius (ff = $\sqrt{r}$)

```
56
57          geodesic_shooter<Oscilloton> pewpew;
58
59          pewpew.shoot(initial_data,0.25,100,null_geodesic);
60
```

AHFinder – Extra notes

- Finding other geometries (e.g. cylinder, 2D AHs like a string, etc.)
- Finding other surfaces (possible to find e.g. chi=0.3 surface)
- ‘Postprocessing_tools’ repo
  (if you want to use a checkpoint to find the AH after running)
- Other cools stuff



Example of AHFinder working in 2D with a weird shape (higher dimensional Black String)

Image credit to Chenxia Gu